# ELECTRONIC (SAMPLE ASSIGNMENT)

Assignment Pedia
Complexity Simplified

**Problem  Design a PID controller**

We are trying to control an electrostatic electrode actuator with a lumped mass of 1 μg, a resonant frequency of 50 kHz, and a Q of 10. We will model the linearized system as a spring-mass-damper 2nd-order system. The plant transfer function $H(s)$ will be $X(s)/F(s)$.

a. First, recast the transfer function in terms of the non-dimensional complex frequency $\hat{s} = s/\omega_0$.

b. Now, assume that you are using a single-pole controller of the form in equation (15.14), with non-dimensionalized time constant $\hat{\tau} = 0.1$. Using a root-locus plot for the overall control system transfer function $X_{out}(s)/X_{in}(s)$, determine the maximum controller gain ($K_0$) at which the system is stable.

c. Instead of using a single-pole controller, design a PID controller that achieves overall critically damped system response with no DC error. Demonstrate results with SIMULINK or MATLAB simulations.

**Problem 1  Design of a PID controller**

a). From Eq. (15.5), the transfer function after normalizing the frequency to the undamped resonant frequency is,

$$\frac{X(s)}{F(s)} = H(s) = \frac{1/k}{\hat{s}^2 + \frac{1}{Q}\hat{s} + 1},$$

where we have,

$$\hat{s} = \frac{s}{\omega_o}, \ where\ \omega_o = 2\pi \cdot 50kHz = \pi \times 10^5\, rad/s$$

$$k = \omega_o^2 m = \pi^2 \times 10^{10}\left(rad/s\right) \times 10^{-9}\left(kg\right) = 10\pi^2$$

$$Q = 10$$

And hence,

$$H(\hat{s}) = \frac{1}{10\pi^2\left(\hat{s}^2 + 0.1\hat{s} + 1\right)}$$

b). We will use a single pole controller, which has the form

$$K(\hat{s}) = \frac{K_o}{1 + 0.1\hat{s}}$$

The loop transfer function is hence,

$$H(\hat{s})K(\hat{s}) = \frac{K_o/10\pi^2}{\left(0.1\hat{s} + 1\right)\left(\hat{s}^2 + 0.1\hat{s} + 1\right)}$$

Using Matlab command rlocus(sys), the root locus plot of the closed loop transfer function is shown in Figure 1 below. A zoomed-in view of the root near the imaginary axis as shown Figure 2 reveals that the maximum gain $K_o$ at which the two poles will coincide with the imaginary axis and hence the system will become unstable is 98.9. The Matlab code used to generate these two plots is provided.

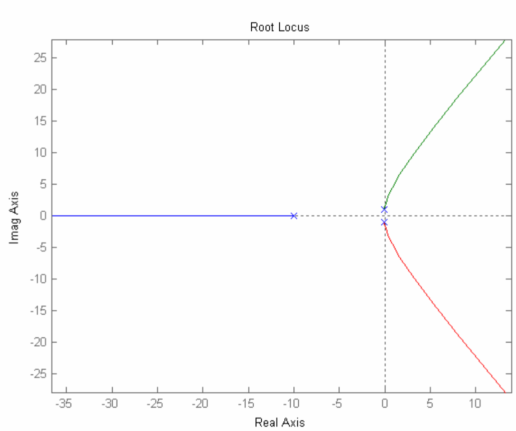



Figure 1. Root locus plot of the closed loop transfer function.

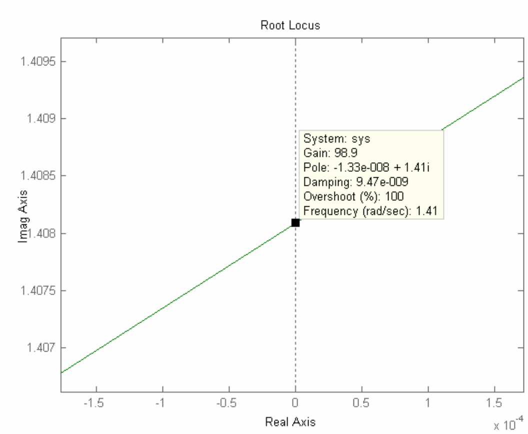



Figure 2. Zoomed-in view near the imaginary axis.

```
% root locus plot

clear all;
close all;

Hden=[1 0.1 1];
Hnum=[1/10/pi^2];
H=tf(Hnum,Hden)

Kden=[0.1 1];
Knum=[1];
K=tf(Knum,Kden);

sys=H*K;

rlocus(sys);
```

c). Instead of using a single-pole controller, we will design a PID controller that achieves overall critically damped system. A normalized PID controller has the form,

$$K(\hat{s}) = K_o\left(1+\frac{\beta}{\hat{s}}+\gamma\hat{s}\right)$$

The closed loop transfer function now becomes,

$$\frac{X_{out}}{X_{in}} = \frac{\frac{K_o}{k}\left(\gamma\hat{s}^2+\hat{s}+\beta\right)}{\hat{s}^3+\left(\frac{1}{Q}+\frac{K_o\gamma}{k}\right)\hat{s}^2+\left(1+\frac{K_o}{k}\right)\hat{s}+\frac{K_o\beta}{k}}$$

This is a third order system with 3 poles and 2 zeros. We know that for a second order system, critically damped response means that the system transfer function has two real poles that are equal, and as a result, the system achieves steady state with the fastest response without overshoot. In order for the system to exhibit a second order behavior, we ideally would want to have 2 equal real poles and a third pole that would cancel out a zero. It turns out that for this problem, it is not possible to achieve the critical damping with a PI controller. What we can do, however, is to have a third pole that is much larger than the 2 real poles such that its fast response does not have much noticeable effect on the second order system behavior. Therefore, we can express the denominator as,

$$\left(\hat{s}+a\right)\left(\hat{s}+b\right)^2, \text{ where } a \gg b$$

And by comparison, we have,

$$\begin{cases} \frac{1}{Q}+\frac{K_o\gamma}{k} = a+2b \\ 1+\frac{K_o}{k} = b^2+2ab \\ \frac{K_o}{k}\beta = ab^2 \end{cases}$$

There could be many choices for a and b that could satisfy the criteria of critical damping, and without further specification on the rise time, for example, we will choose one pair that works. From part a), we can derive that the plant $H(s)$ has two complex poles $0.05 \pm j$. In order to have faster response, we want to have two real poles move to the left of the S-plane. Let's choose $b$=1, and $a$=1000, say. We can derive that,

$$\begin{cases} K_o = 2000k = 2\times 10^4\pi^2 \\ \beta = 0.5 \\ \gamma = 0.501 \end{cases}$$

Hence the PID controller that is chosen is,

$$K(\hat{s}) = 2\times10^4\pi^2\left(1+\frac{0.5}{\hat{s}}+0.501\hat{s}\right)$$

The root locus plot of the new loop transfer function

$$H(\hat{s})K(\hat{s}) = \frac{2\times10^3\left(1+\frac{0.5}{\hat{s}}+0.501\hat{s}\right)}{\left(\hat{s}^2+0.1\hat{s}+1\right)}$$

is shown in Figure 3. The two zeros are very close to the real poles. Although it is advantageous to have the zeros far away from the dominant poles, since the zeros of a system affect pretty much the amplitude, rather than the oscillation nature of the system (as long as they are in the negative half of the s-plane), we do not have to worry about them too much. The step response of the overall closed loop system is shown in Figure 4, demonstrating the critical damped response, with a rise time on the order of 3 ms.

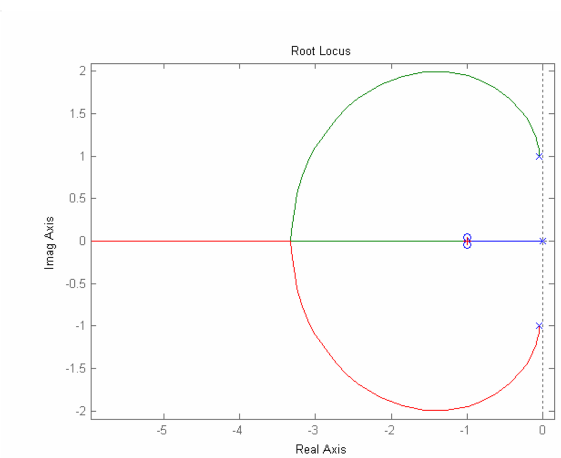



Figure 3. Root locus plot of the system with PID controller.

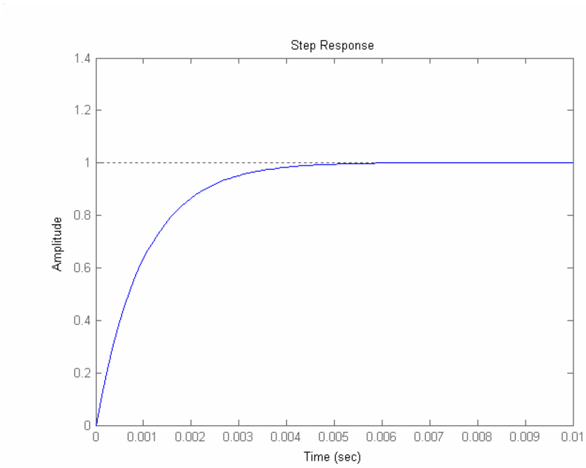



Figure 4. Step response of the system wit PID controller.

The MATLAB coding used for this portion of the problem is provided below:

```
% root locus plot

clear all;
close all;

Hden=[1 0.1 1];
Hnum=[1/10/pi^2];
H=tf(Hnum,Hden)

Kden=[1 0];
ko=20000*pi^2;
Knum=ko*[0.501 1 0.5];
K=tf(Knum,Kden);
sys=H*K
rlocus(sys);
figure
step(sys/(1+sys),0.01)
```

**Problem Stability and capacitive loads**

We are often faced with the task of driving capacitive loads including electrostatic transducers such as parallel-plate actuators. Such loads, when driven at high frequencies, can create circuit instabilities without proper care. In this example, assume we are driving a capacitive load of $C$=200 pF with a buffer op-amp controller $K(s)$ (figure, A).

(A) (B)

a. Assume that the amplifier (like ALL amplifiers) has a finite output resistance $R_0$=40 Ω, using th circuit model in (B), with the load capacitor $C$, determine analytically the loop transmission function $H(s)K(s)$ for the circuit, where the loop transmission function is defined as $V_0 = H(s)K(s)(V - V_0)$.

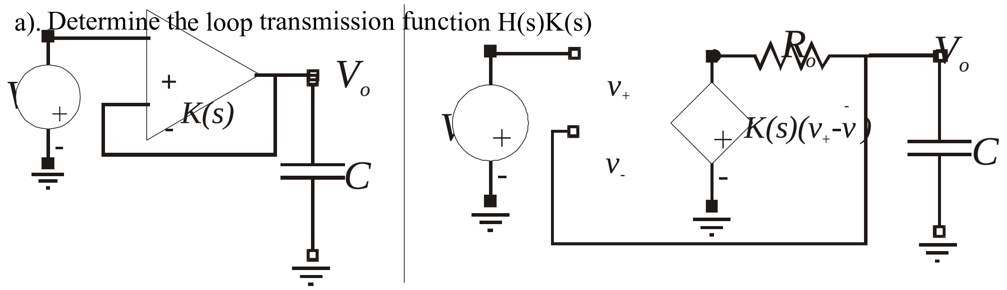
b. Now assume that $K(s)$ is a 2$^{nd}$-order transfer function of the form

$$K(s) = \frac{10^{3.5}}{\left(1+\frac{s}{2\pi\cdot10^5}\right)\left(1+\frac{s}{2\pi\cdot10^8}\right)}$$

which approximates the National Semiconductor LM359 high-speed op-amp. Determine the overall loop transmission function $H(s)K(s)$. Make a Bode plot for the loop transmission function and determine the phase margin with and without the capacitor. What is the maximum capacitive load that this amplifier can drive and be stable?

**Problem** **Stability and capacitive loads**

a). Determine the loop transmission function H(s)K(s)




(A) (B)

From the circuit model on the left, we have

$$V_- = V_0$$
$$V_+ = V$$

Applying KCL at the node connecting the output resistor and the capacitor:

$$\frac{V_0 - K_s(V - V_0)}{R_0} + \frac{V_0}{\frac{1}{sC}} = 0$$

Rearrange terms, we get,

$$V_o = \frac{K(s)}{1 + R_o \cdot s \cdot C} \cdot (V - V_0)$$

The model is equivalent to a linear feedback system with an overall transfer function as,

$$V_0 = H(s)K(s)(V - V_0)$$

which is referred to as Black's formula.

The Loop transmission function is hence,

$$H(s)K(s) = \frac{K(s)}{1 + R_o \cdot s \cdot C}$$

b). Assuming

$$K(s) = \frac{10^{3.5}}{\left(1 + s/2\pi \times 10^5\right)\left(1 + s/2\pi \times 10^8\right)}$$

The transmission function then becomes

$$H(s)K(s) = \frac{10^{3.5}}{\left(1 + s/2\pi \times 10^5\right)\left(1 + s/2\pi \times 10^8\right)} \cdot \frac{1}{1 + R_o \cdot s \cdot C}$$

$$= \frac{3162.28\pi^2}{\pi^2 + s\left(0.5005\pi \times 10^{-5} + R_o \cdot C \cdot \pi^2\right) + s^2\left(0.25 \times 10^{-13} + 0.5005 \times 10^{-5} \cdot R_o \cdot C \cdot \pi\right) + s^3\left(0.25 \times 10^{-13} \cdot R_o \cdot C\right)}$$

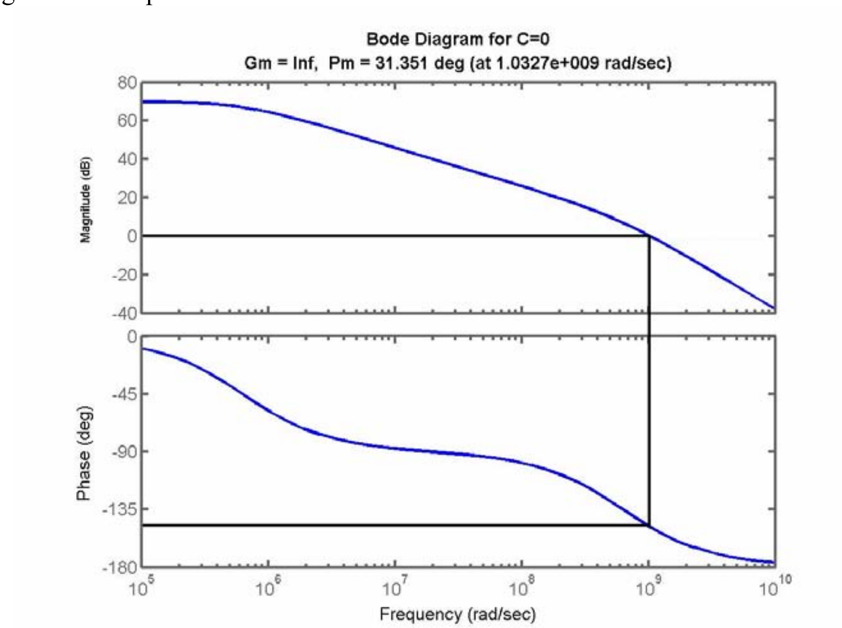
For the case C = 0, the loop transmission function becomes

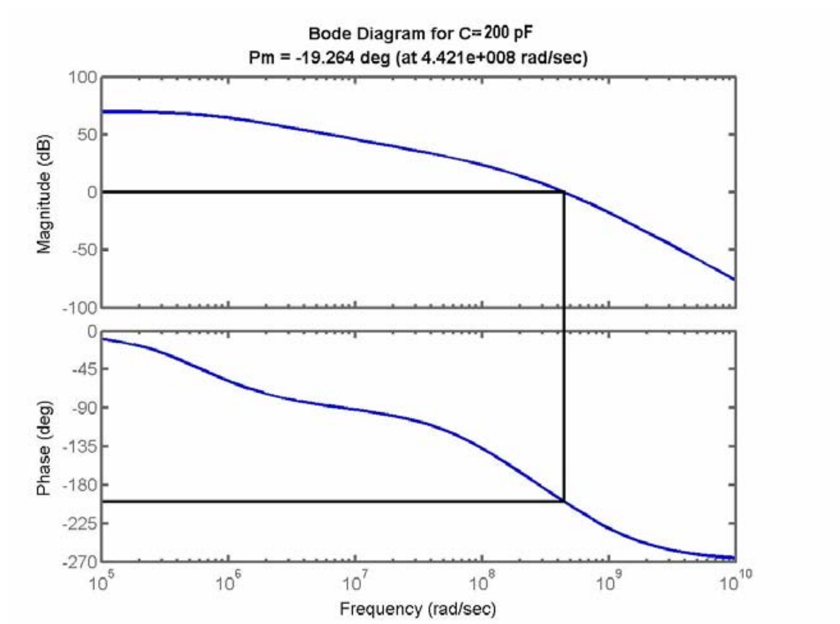
$$H(s)K(s) = \frac{3162.28\pi^2}{\pi^2 + s \cdot \left(0.5005\pi \times 10^{-5}\right) + s^2\left(0.25 \times 10^{-13}\right)}$$

For the case $C = 200 \times 10^{-9}$ F, the loop transmission function becomes

$$H(s)K(s) = \frac{3162.28 \cdot \pi^2}{\pi^2 + s\left(1.58 \times 10^{-5}\right) + s^2\left(1.5079 \times 10^{-13}\right) + s^3\left(2 \times 10^{-22}\right)}$$

The phase margin and bode plots of the two cases are shown below.




The phase margin angle γ is defined as

$$\gamma = 180^{\circ} + \alpha$$

where α is the phase angle where the amplitude of the output signal is equal to the amplitude of the input signal. A system is stable implies a positive phase margin value.

For the capacitance value of 0 F, the phase margin is $31.351^0$.

For the case with C = 200 pF = $200\text{x}10^{-12}$ F the phase margin is $-19.264^0$. The system is unstable. We can see that a larger capacitance implies a smaller phase margin.

The phase margin is zero when the capacitance is decreased to about 18.44 pF, which is the maximum capacitance to be driven stably and the corresponding frequency is 147 MHz.